1. Zkrať lomené výrazy:

a) $\frac{72abx}{84aby}=\frac{8\cdot 9x}{4\cdot 21y}=\frac{2\cdot 3x}{7y}=\frac{6x}{7y}$

b) $\frac{p^2-2pq+q^2}{p^2-q^2}=\frac{(p-q)^2}{p^2-q^2}=\frac{(p-q)(p-q)}{(p-q)(p+q)}=\frac{p-q}{p+q}$

c) $\frac{4a^2-1}{4a^2-4a+1}=\frac{(2a)^2-1}{(2a)^2-4a+1}=\frac{(2a+1)(2a-1)}{(2a-1)^2}=\frac{2a+1}{2a-1}$

d) $\frac{16-8a+a^2}{ab-4b}=\frac{4^2-8a+a^2}{b(a-4)}=\frac{(4-a)^2}{b(a-4)}=\frac{(4-a)^2}{-b(4-a)}=\frac{4-a}{-b}=\frac{a-4}{b}$

e) $\frac{a^2+2ab+b^2-c^2}{a^2+2ac+c^2-b^2}=\frac{(a+b)^2-c^2}{(a+c)^2-b^2}=\frac{[(a+b)-c][(a+b)+c]}{[(a+c)-b][(a+c)+b]}=\frac{a+b-c}{a+c-b}$

f) $\frac{ab+2b-ac-2c}{ab-2b-ac+2c}=\frac{a(b-c)+2(b-c)}{a(b-c)-2(b-c)}=\frac{(b-c)(a+2)}{(b-c)(a-2)}=\frac{a+2}{a-2}$

g) $\frac{x^2-4x+4}{x^2-5x+6}=\frac{(x-2)^2}{(x-2)(x-3)}=\frac{x-2}{x-3}$

h) $\frac{3uv+9v-2u-6}{3uv-2u-9v+6}=\frac{3v(u+3)-2(u+3)}{u(3v-2)-3(3v-2)}=\frac{(u+3)(3v-2)}{(3v-2)(u-3)}=\frac{u+3}{u-3}$

i) $\frac{a^2+2a-15}{3a+15}=\frac{(a+5)(a-3)}{3(a+5)}=\frac{a-3}{3}$

j) $\frac{a^2-a-20}{a^2+a-30}=\frac{(a-5)(a+4)}{(a+6)(a-5)}=\frac{a+4}{a+6}$

k) $\frac{3x^2+x-10}{4x^2+x-14}=\frac{3x^2+6x-5x-10}{4x^2+8x-7x-14}=\frac{3x(x+2)-5(x+2)}{4x(x+2)-7(x+2)}=\frac{(x+2)(3x-5)}{(x+2)(4x-7)}=\frac{3x-5}{4x-7}$

l) $\frac{x^3+x^2y+xy^2}{x^3y-y^4}=\frac{x(x^2+xy+y^2)}{y(x^3-y^3)}=\frac{x(x^2+xy+y^2)}{y(x-y)(x^2+xy+y^2)}=\frac{x}{y(x-y)}$

2. Rozšiř dané lomené výrazy na tvary se jmenovatelem ve složené závorce:

a) $\frac{x}{y-2}\quad \{(2-y)\}$

$\frac{x}{y-2}\cdot\frac{-1}{-1}=\frac{-x}{2-y}$

b) $\frac{x+3}{2-x}\quad \{(x^2-4)\}$

$\frac{x+3}{2-x}\cdot\frac{-1}{-1}=\frac{(-x-3)}{x-2}\cdot\frac{x+2}{x+2}=\frac{-x^2-5x-6}{x^2-4}$

c) $\frac{y-1}{x+1}\quad \{(x^3+1)\}$

$\frac{y-1}{x+1}=\frac{y-1}{x+1}\cdot\frac{x^2-x+1}{x^2-x+1}=\frac{(y-1)(x^2-x+1)}{x^3+1}$

d) $\frac{a+1}{(a+1)^2}\quad \{(a^2-1)\}$

$\frac{a+1}{(a+1)^2}=\frac{1}{a+1}\cdot\frac{a-1}{a-1}=\frac{a-1}{a^2-1}$

e) $\dfrac{x-1}{x^2-1}\quad \left\{(x+1)^3\right\}$

$$\frac{x-1}{x^2-1}=\frac{x-1}{(x-1)(x+1)}=\frac{1}{x+1}\cdot\frac{(x+1)^2}{(x+1)^2}=\frac{x^2+2x+1}{(x+1)^3}$$

3. Dokaž, že platí:

a) $\dfrac{ac+bx+ax+bc}{ay+2bx+2ax+by}=\dfrac{ac+ax+bx+bc}{ay+by+2bx+2ax}=\dfrac{a(c+x)+b(x+c)}{y(a+b)+2x(b+a)}=\dfrac{(x+c)(a+b)}{(a+b)(y+2x)}=\dfrac{x+c}{2x+y}$

b) $\dfrac{x-xy+z-zy}{1-3y+3y^2-y^3}=\dfrac{x(1-y)+z(1-y)}{1-y^3-3y+3y^2}=\dfrac{(1-y)(x+z)}{(1-y)(1+y+y^2)-3y(1-y)}=\dfrac{(1-y)(x+z)}{(1-y)\left[(1+y+y^2)-3y\right]}=$

$=\dfrac{(x+z)}{(1-2y+y^2)}=\dfrac{x+z}{(1-y)^2}$

c) $\dfrac{3a^3+ab^2-6a^2b-2b^3}{9a^5-ab^4-18a^4b+2b^5}=\dfrac{3a^3-6a^2b+ab^2-2b^3}{9a^5-18a^4b-ab^4+2b^5}=\dfrac{3a^2(a-2b)+b^2(a-2b)}{9a^4(a-2b)-b^4(a-2b)}=\dfrac{(a-2b)(3a^2+b^2)}{(a-2b)(9a^4-b^4)}=$

$=\dfrac{3a^2+b^2}{(3a^2)^2-(b^2)^2}=\dfrac{3a^2+b^2}{(3a^2+b^2)(3a^2-b^2)}=\dfrac{1}{3a^2-b^2}$

4. Uprav a uveď podmínky, za kterých mají výrazy smysl:

a) $\dfrac{2m-n}{m-n}+\dfrac{m}{n-m}=\dfrac{2m-n}{m-n}+\dfrac{m}{-(m-n)}=\dfrac{2m-n-m}{m-n}=\dfrac{m-n}{m-n}=1$

$m\neq n$

b) $\dfrac{x-y}{xy}-\dfrac{z-y}{zy}+\dfrac{x+z}{xz}=\dfrac{xz-yz}{xyz}-\dfrac{xz-xy}{xyz}+\dfrac{xy+yz}{xyz}=\dfrac{xz-yz-xz+xy+xy+yz}{xyz}=\dfrac{2xy}{xyz}=\dfrac{2}{z}$

$x\neq 0; y\neq 0; z\neq 0$

c) $\dfrac{u^2+1}{u+1}-u=\dfrac{u^2+1}{u+1}-\dfrac{u(u+1)}{u+1}=\dfrac{u^2+1-u^2-u}{u+1}=\dfrac{1-u}{u+1}$

$u\neq -1$

d) $\dfrac{4mn}{m-n}+(m-n)=\dfrac{4mn}{m-n}+\dfrac{(m-n)(m-n)}{(m-n)}=\dfrac{4mn+m^2-2mn+n^2}{m-n}=\dfrac{m^2+2mn+n^2}{m-n}=\dfrac{(m+n)^2}{m-n}$

$m\neq n$

e) $1-\dfrac{2p}{q}+\dfrac{p^2}{q^2}-\dfrac{(q-p)^2}{q^2}=\dfrac{q^2}{q^2}-\dfrac{2pq}{q^2}+\dfrac{p^2}{q^2}-\dfrac{q^2-2qp+p^2}{q^2}=\dfrac{q^2-2pq+p^2-(q^2-2qp+p^2)}{q^2}=0$

$q\neq 0$

f) $\dfrac{7v-1}{2v^2+6v}+\dfrac{5-3v}{v^2-9}=\dfrac{7v-1}{2v(v+3)}+\dfrac{5-3v}{(v-3)(v+3)}=\dfrac{(7v-1)(v-3)}{2v(v+3)(v-3)}+\dfrac{2v(5-3v)}{2v(v-3)(v+3)}=$

$=\dfrac{7v^2-21v-v+3+10v-6v^2}{2v(v-3)(v+3)}=\dfrac{v^2-12v+3}{2v(v-3)(v+3)}$

$v\neq 0; v\neq \pm 3$

g) $\dfrac{2p+q}{p^2+pq}+\dfrac{1}{p}-\dfrac{1}{p+q}=\dfrac{2p+q}{p(p+q)}+\dfrac{p+q}{p(p+q)}-\dfrac{p}{p(p+q)}=\dfrac{2p+q+p+q-p}{p(p+q)}=\dfrac{2(p+q)}{p(p+q)}=\dfrac{2}{p}$

$p\neq 0; p\neq -q$

$$\frac{a-2b}{a+b}-\frac{2a-b}{b-a}-\frac{2a^2}{a^2-b^2}=\frac{a-2b}{a+b}-\frac{2a-b}{-(a-b)}-\frac{2a^2}{(a-b)(a+b)}=$$

h) $$=\frac{(a-2b)(a-b)}{(a+b)(a-b)}+\frac{(2a-b)(a+b)}{(a-b)(a+b)}+\frac{-2a^2}{(a-b)(a+b)}=$$

$$=\frac{a^2-ab-2ab+2b^2+2a^2+2ab-ab-b^2-2a^2}{(a-b)(a+b)}=\frac{a^2-2ab+b^2}{(a-b)(a+b)}=\frac{(a-b)^2}{(a-b)(a+b)}=\frac{a-b}{a+b}$$

$a\neq\pm b$

$$\frac{1+x}{1-x}-\frac{1-x}{1+x}-\frac{x(4-x)}{1-x^2}=\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{-x(4-x)}{(1-x)(1+x)}=$$

i) $$=\frac{(1+x)^2}{(1-x)(1+x)}-\frac{(1-x)^2}{(1+x)(1-x)}+\frac{-x(4-x)}{(1-x)(1+x)}=\frac{1+2x+x^2-(1-2x+x^2)-4x+x^2}{(1-x)(1+x)}=$$

$$=\frac{x^2}{1-x^2}$$

$x\neq\pm1$

$$\frac{4}{3m-3n}-\frac{3m-4n}{2m^2-4mn+2n^2}=\frac{4}{3(m-n)}-\frac{3m-4n}{2(m^2-2mn+n^2)}=\frac{4\cdot2(m-n)}{2\cdot3(m-n)^2}-\frac{3(3m-4n)}{3\cdot2(m-n)^2}=$$

j) $$=\frac{8m-8n-(9m-12n)}{2\cdot3(m-n)^2}=\frac{4n-m}{6(m-n)^2}$$

$m\neq n$

5. Zjednoduš výraz $a+1+\frac{a-1}{a^2-a+1}$ a dokaž, že má smysl pro každé a.

$$a+1+\frac{a-1}{a^2-a+1}=\frac{(a+1)(a^2-a+1)}{a^2-a+1}+\frac{a-1}{a^2-a+1}=\frac{a^3-a^2+a+a^2-a+1+a-1}{a^2-a+1}=\frac{a^3+a}{a^2-a+1}$$

Zlomek má smysl vždy, když je jeho jmenovatel různý od nuly. Mnohočlen ve jmenovateli se však nule nikdy nerovná.

6. Dokaž, že pro všechna nenulová čísla t je součet výrazů $1+t$ a $1+\frac{1}{t}$ roven jejich součinu.

$$1+t+1+\frac{1}{t}=2+t+\frac{1}{t}=\frac{(2+t)t}{t}+\frac{1}{t}=\frac{t^2+2t+1}{t}=\frac{(t+1)^2}{t}$$

$$(1+t)\left(1+\frac{1}{t}\right)=(1+t)\frac{t+1}{t}=\frac{(t+1)^2}{t}$$

7. Uprav a uveď podmínky, za kterých mají výrazy smysl:

a) $$\frac{3x^3b^3}{25y^4}\cdot\left(-\frac{15y}{b^2}\right)=-\frac{3x^3b^3}{25y^4}\cdot\frac{15y}{b^2}=-\frac{9x^3b}{5y^3}$$

$y\neq0; b\neq0$

b) $$\frac{9x}{a^3}\cdot\left(-\frac{y}{32b^2}\right)\left(-\frac{4a}{27xy}\right)\cdot24a^2b^3=\frac{3^2x}{a^3}\cdot\frac{y}{2^5b^2}\cdot\frac{2^2a}{3^3xy}\cdot2^3\cdot3a^2b^3=b$$

$x\neq0; y\neq0; a\neq0; b\neq0$

c) $$\frac{2v^2+8v+8}{v-2}\cdot\frac{(v-2)^2}{4(v+2)}=\frac{2(v^2+4v+4)}{1}\cdot\frac{(v-2)}{4(v+2)}=(v+2)^2\cdot\frac{(v-2)}{2(v+2)}=(v+2)\cdot\frac{(v-2)}{2}=\frac{v^2-4}{2}$$

$v\neq\pm2$

d) $\left(\frac{1}{a}-\frac{1}{b}\right)\cdot\frac{a^2}{a-b}=\left(\frac{b}{ab}-\frac{a}{ab}\right)\cdot\frac{a^2}{a-b}=\frac{b-a}{ab}\cdot\frac{a^2}{a-b}=-\frac{a-b}{b}\cdot\frac{a}{a-b}=-\frac{a}{b}$

$a\neq 0; b\neq 0; a\neq b$

e) $\left(\frac{3}{1+s}-1\right)\left(\frac{3}{2-s}-1\right)=\left(\frac{3}{1+s}-\frac{1(1+s)}{1+s}\right)\left(\frac{3}{2-s}-\frac{1(2-s)}{2-s}\right)=\left(\frac{2-s}{1+s}\right)\left(\frac{1+s}{2-s}\right)=1$

$s\neq -1; s\neq 2$

$\left(y+1+\frac{1}{2y-1}\right)\left(y-1+\frac{1}{2y+1}\right)=\left[\frac{(y+1)(2y-1)}{2y-1}+\frac{1}{2y-1}\right]\left[\frac{(y-1)(2y+1)}{2y+1}+\frac{1}{2y+1}\right]=$

f) $=\left(\frac{2y^2-y+2y-1+1}{2y-1}\right)\left(\frac{2y^2+y-2y-1+1}{2y+1}\right)=\left(\frac{2y^2+y}{2y-1}\right)\left(\frac{2y^2-y}{2y+1}\right)=\frac{y(2y+1)}{2y-1}\cdot\frac{y(2y-1)}{2y+1}=y^2$

$y\neq\pm\frac{1}{2}$

$\left(\frac{x^2}{x-y}-x\right)\left(\frac{x^2}{y^2}-\frac{y}{x}\right)=\left(\frac{x^2}{x-y}-\frac{x(x-y)}{x-y}\right)\left(\frac{x^2}{y^2}\cdot\frac{x}{x}-\frac{y}{x}\cdot\frac{y^2}{y^2}\right)=\left(\frac{x^2-x^2+xy}{x-y}\right)\left(\frac{x^3-y^3}{xy^2}\right)=$

g) $=\frac{xy}{x-y}\cdot\frac{(x-y)(x^2+xy+y^2)}{xy^2}=\frac{x^2+xy+y^2}{y}$

$x\neq 0; y\neq 0; x\neq y$

$\left[\frac{3}{(x-3)^3}+\frac{1}{x-3}-\frac{3}{x^2-9}\right]\cdot\frac{x^2-6x+9}{x^2+9}=\left[\frac{3}{(x-3)^3}+\frac{1}{x-3}-\frac{3}{(x-3)(x+3)}\right]\cdot\frac{(x-3)^2}{x^2+9}=$

$=\left[\frac{3}{(x-3)^3}+\frac{1}{x-3}-\frac{3}{(x-3)(x+3)}\right]\cdot\frac{(x-3)^2}{x^2+9}=\left[\frac{3(x+3)+1(x-3)^2(x+3)-3(x-3)^2}{(x-3)^3(x+3)}\right]\cdot\frac{(x-3)^2}{x^2+9}=$

h) $=\left[\frac{3x+9+(x^2-6x+9)(x+3)-3(x^2-6x+9)}{(x-3)^3(x+3)}\right]\cdot\frac{(x-3)^2}{x^2+9}=$

$=\left[\frac{3x+9+x^3-6x^2+9x+3x^2-18x+27-3x^2+18x-27}{(x-3)^3(x+3)}\right]\cdot\frac{(x-3)^2}{x^2+9}=$

$=\frac{x^3-6x^2+12x+9}{(x-3)^3(x+3)}\cdot\frac{(x-3)^2}{x^2+9}=\frac{x^3-6x^2+12x+9}{(x-3)(x+3)}\cdot\frac{1}{x^2+9}=\frac{x^3-6x^2+12x+9}{(x^2-9)(x^2+9)}=\frac{x^3-6x^2+12x+9}{x^4-81}$

$x\neq\pm 3$

8. Uprav a uveď podmínky, za kterých mají výrazy smysl:

a) $\left(\frac{18a^2}{b^3}\cdot\frac{c}{2a^3}\right):\left(-\frac{3a}{b^2c}\right)=-\frac{18a^2}{b^3}\cdot\frac{c}{2a^3}\cdot\frac{b^2c}{3a}=-\frac{3c^2}{a^2b}$

$a\neq 0; b\neq 0; c\neq 0$

b) $\frac{a^2+ax}{x-x^2}:\frac{x^2+ax}{a-ax}=\frac{a(a+x)}{x(1-x)}\cdot\frac{a(1-x)}{x(x+a)}=\left(\frac{a}{x}\right)^2$

$x\neq 0; a\neq 0; x\neq 1; x\neq -a$

$\left(\frac{2x^2-4x+2}{x^2+1}:\frac{6x-6}{x^4-1}\right):\frac{x+1}{3}=\frac{2(x^2-2x+1)}{x^2+1}\cdot\frac{(x^2)^2-1}{6(x-1)}\cdot\frac{3}{x+1}=\frac{(x-1)^2}{x^2+1}\cdot\frac{(x^2+1)(x^2-1)}{(x-1)(x+1)}=$

c) $=(x-1)^2$

$x\neq\pm 1$

d)
$$\frac{r^4-s^4}{r^2s^2}:\left[\left(1+\frac{s^2}{r^2}\right)\left(1-\frac{2r}{s}+\frac{r^2}{s^2}\right)\right]=\frac{\left(r^2-s^2\right)\left(r^2+s^2\right)}{r^2s^2}:\left[\left(\frac{r^2+s^2}{r^2}\right)\left(\frac{s^2-2rs+r^2}{s^2}\right)\right]=$$
$$=\frac{\left(r^2-s^2\right)\left(r^2+s^2\right)}{r^2s^2}:\left(\frac{r^2+s^2}{r^2}\cdot\frac{\left(s-r\right)^2}{s^2}\right)=\frac{\left(r^2-s^2\right)\left(r^2+s^2\right)}{r^2s^2}\cdot\frac{r^2s^2}{\left(r^2+s^2\right)\left(s-r\right)^2}=$$
$$=\frac{\left(r^2-s^2\right)}{\left(s-r\right)^2}=\frac{\left(r-s\right)\left(r+s\right)}{\left[\left(-1\right)^2\left(r-s\right)^2\right]}=\frac{r+s}{r-s}$$
$r\neq 0; s\neq 0; r\neq s$

e)
$$\left(1+\frac{a^3}{b^3}\right):\left(1+\frac{a}{b}\right)=\left(\frac{b^3+a^3}{b^3}\right):\left(\frac{b+a}{b}\right)=\frac{\left(b+a\right)\left(b^2-ba+a^2\right)}{b^3}\cdot\frac{b}{b+a}=\frac{b^2-ba+a^2}{b^2}$$
$b\neq 0; b\neq -a$

f)
$$\left(c^3-d^3\right):\left(c+\frac{d^2}{c+d}\right)=\left(c-d\right)\left(c^2+cd+d^2\right):\left(c\frac{c+d}{c+d}+\frac{d^2}{c+d}\right)=\left(c-d\right)\left(c^2+cd+d^2\right):\left(\frac{c^2+cd+d^2}{c+d}\right)=$$
$$=\left(c-d\right)\left(c^2+cd+d^2\right)\cdot\frac{c+d}{c^2+cd+d^2}=\left(c-d\right)\left(c+d\right)=c^2-d^2$$
$c\neq -d$

9. Uprav a uveď podmínky, za kterých mají výrazy smysl:

a)
$$\frac{m-\frac{4}{m}}{m+2}=\frac{\frac{m^2-4}{m}}{m+2}=\frac{m^2-4}{m\left(m+2\right)}=\frac{\left(m-2\right)\left(m+2\right)}{m\left(m+2\right)}=\frac{\left(m-2\right)}{m}$$
$m\neq 0; m\neq -2$

b)
$$\frac{\frac{a+b}{a-b}-1}{\frac{a+b}{a-b}+1}=\frac{\frac{a+b-\left(a-b\right)}{a-b}}{\frac{a+b+\left(a-b\right)}{a-b}}=\frac{\frac{2b}{a-b}}{\frac{2a}{a-b}}=\frac{2b\left(a-b\right)}{2a\left(a-b\right)}=\frac{b}{a}$$
$a\neq 0; a\neq -b$

c)
$$\frac{\frac{x}{4}-\frac{x-1}{5}}{\frac{x+1}{6}-\frac{x-1}{10}}=\frac{\frac{5x-4\left(x-1\right)}{4\cdot 5}}{\frac{5\left(x+1\right)-3\left(x-1\right)}{30}}=\frac{\frac{x+4}{20}}{\frac{2x+8}{30}}=\frac{30\left(x+4\right)}{20\left(2x+8\right)}=\frac{3\left(x+4\right)}{2\cdot 2\left(x+4\right)}=\frac{3}{4}$$
$x\neq -4$

d)
$$\frac{\frac{r+s}{r-s}-\frac{r-s}{r+s}}{1-\frac{r^2+s^2}{r^2-s^2}}=\frac{\frac{\left(r+s\right)^2-\left(r-s\right)^2}{\left(r-s\right)\left(r+s\right)}}{\frac{r^2-s^2-\left(r^2+s^2\right)}{r^2-s^2}}=\frac{\frac{r^2+2rs+s^2-\left(r^2-2rs+s^2\right)}{r^2-s^2}}{\frac{-2s^2}{r^2-s^2}}=\frac{\frac{4rs}{r^2-s^2}}{\frac{-2s^2}{r^2-s^2}}=\frac{4rs\left(r^2-s^2\right)}{-2s^2\left(r^2-s^2\right)}=-\frac{2r}{s}$$
$s\neq 0; r\neq \pm s$

e)
$$\frac{2-\frac{k^2+z^2}{kz}}{\frac{k}{z^2}-\frac{2}{z}+\frac{1}{k}}=\frac{\frac{2kz-\left(k^2+z^2\right)}{kz}}{\frac{k^2}{z^2}-\frac{2zk}{z^2k}+\frac{z^2}{z^2k}}=\frac{\frac{-\left(k^2-2kz+z^2\right)}{kz}}{\frac{k^2-2kz+z^2}{z^2k}}=\frac{\frac{-\left(k-z\right)^2}{kz}}{\frac{\left(k-z\right)^2}{z^2k}}=-\frac{z^2k\left(k-z\right)^2}{kz\left(k-z\right)^2}=-z$$
$k\neq 0; z\neq 0; k\neq z$

f) $\dfrac{\dfrac{a^2}{b^2}-\dfrac{a}{b}}{\dfrac{a^2+b^2}{ab}-2}:\dfrac{a^2}{b}=\dfrac{\dfrac{a^2-ab}{b^2}}{\dfrac{a^2+b^2-2ab}{ab}}\cdot\dfrac{b}{a^2}=\dfrac{\dfrac{a(a-b)}{b^2}}{\dfrac{(a-b)^2}{ab}}\cdot\dfrac{b}{a^2}=\dfrac{\dfrac{a(a-b)}{b^2}}{\dfrac{(a-b)^2}{ab}}\dfrac{a(a-b)ab}{(a-b)^2b^2}\cdot\dfrac{b}{a^2}=\dfrac{1}{a-b}$

$a\neq 0; b\neq 0; a\neq b$

$$\frac{\dfrac{a}{b}-\dfrac{b^2}{a^2}}{1-\dfrac{b}{a}}\left(a^2-ab+b^2\right)=\frac{\dfrac{a^3-b^3}{a^2b}}{\dfrac{a-b}{a}}\left(a^2-ab+b^2\right)=\frac{\dfrac{(a-b)\left(a^2+ab+b^2\right)}{a^2b}}{\dfrac{a-b}{a}}\left(a^2-ab+b^2\right)=$$

g) $=\dfrac{(a-b)\left(a^2+ab+b^2\right)a}{a^2b(a-b)}\left(a^2-ab+b^2\right)=\dfrac{\left(a^2+ab+b^2\right)\left(a^2-ab+b^2\right)}{ab}=$

$$=\frac{a^4-a^3b+a^2b^2+a^3b-a^2b^2+ab^3+a^2b^2-ab^3+b^4}{ab}=\frac{a^4+a^2b^2+b^4}{ab}$$

$a\neq 0; b\neq 0; a\neq b$

$$\frac{\dfrac{a^4-b^4}{a^2b^2}}{\left(1+\dfrac{b^2}{a^2}\right)\left(1-\dfrac{2a}{b}+\dfrac{a^2}{b^2}\right)}=\frac{\dfrac{\left(a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^2}}{\left(\dfrac{a^2+b^2}{a^2}\right)\left(\dfrac{b^2-2ab+a^2}{b^2}\right)}=\frac{\dfrac{\left(a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^2}}{\dfrac{\left(a^2+b^2\right)}{a^2}\dfrac{(a-b)^2}{b^2}}=$$

h) $=\dfrac{\left(a^2-b^2\right)\left(a^2+b^2\right)a^2b^2}{a^2b^2\left(a^2+b^2\right)(a-b)^2}=\dfrac{\left(a^2-b^2\right)}{(a-b)^2}=\dfrac{(a-b)(a+b)}{(a-b)^2}=\dfrac{a+b}{a-b}$

$a\neq 0; b\neq 0; a\neq b$

$$\frac{\dfrac{x^3}{y^2}+\dfrac{x^2}{y}+x+y}{\dfrac{x^2}{y^2}-\dfrac{y^2}{x^2}}=\frac{\dfrac{x^3}{y^2}+\dfrac{x^2y}{y^2}+\dfrac{xy^2}{y^2}+\dfrac{y^3}{y^2}}{\dfrac{x^4-y^4}{x^2y^2}}=\frac{\dfrac{x^3+x^2y+xy^2+y^3}{y^2}}{\dfrac{x^4-y^4}{x^2y^2}}=\frac{\dfrac{x^2(x+y)+y^2(x+y)}{y^2}}{\dfrac{\left(x^2-y^2\right)\left(x^2+y^2\right)}{x^2y^2}}=$$

i) $=\dfrac{\dfrac{(x+y)\left(x^2+y^2\right)}{y^2}}{\dfrac{\left(x^2-y^2\right)\left(x^2+y^2\right)}{x^2y^2}}=\dfrac{(x+y)\left(x^2+y^2\right)x^2y^2}{\left(x^2-y^2\right)\left(x^2+y^2\right)y^2}=\dfrac{x^2(x+y)}{x^2-y^2}=\dfrac{x^2(x+y)}{(x-y)(x+y)}=\dfrac{x^2}{(x-y)}$

$x\neq 0; y\neq 0; x\neq \pm y$